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Two-loop $\mathcal{O}(\alpha_s^2)$ MSSM corrections to the pole masses of heavy quarks

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Abstract. We present the results for two-loop MSSM corrections to the relation between pole and running masses of heavy quarks up to the order $\mathcal{O}(\alpha_s^2)$. The running masses are defined in the $\overline{\mathrm{DR}}$ scheme, usually used in multiloop calculations in supersymmetric theories. The analysis of the values of these corrections in different regions of the parameter space is provided.

1 Introduction

Recently a lot of theoretical and experimental efforts have been spent to find some new physics beyond that described by the standard model (SM). One of the most popular ways to extend SM is to make our world supersymmetric at some higher scale than those currently tested by experimental facilities over the world. A way of doing this minimally and at the same time to have renormalizable local quantum field theory is to consider the $N = 1$ SUSY field theory. This is obtained from SM by replacing SM fields with groups of fields, forming representations of the $N = 1$ SUSY algebra, and introducing an additional Higgs doublet required by supersymmetry. This way we obtain the minimal supersymmetric standard model (MSSM) [1–3].

Huge theoretical research in this direction was made in the last few years; also the parameter space of this model was heavily constrained by experimentalists. Today people consider radiative SUSY corrections to SM precision observables and try to observe SUSY particles directly in an experiment. The precision with which masses of heavy quarks are known at present requires that higher order corrections, in addition to the leading one-loop SUSY corrections, be taken into account.

It is the aim of the present article to calculate the "leading" $\mathcal{O}(\alpha_s^2)$ MSSM corrections to the pole masses of heavy quarks and to see how they affect the SUSY particle spectra obtained from the renormalization group analysis with universal boundary conditions at the GUT scale. In fact we will be interested in the relation between the pole mass of a heavy quark and the running heavy quark $\overline{\mathrm{DR}}$ mass. The latter is defined as a quantity computed in dimensional reduction and renormalized minimally[1]. By "leading" corrections we mean those, which are mediated by strong interactions; formally they should be dominant. However, as is known from one-loop calculations, in the case of the b-quark there exists a large contribution from a stop–chargino loop enhanced by the t-quark Yukawa coupling, $\tan\beta$ or by the supersymmetric Higgs mass parameter μ. So, in this sense, our results for the b-quark are not complete. But they can be used to see the typical size of these corrections and already draw some conclusions on whether they should be accounted for in an accurate analysis or not.

To compute the needed diagrams we use the large mass expansion procedure and for the moment we restrict ourselves only to terms up to the $\mathcal{O}(m_b^2/M_{\mathrm{SUSY}}^2)$ order in the case of the b-quark and up to the $\mathcal{O}(m_t^2/M_{\mathrm{SUSY}}^2)$ order in the case of the t-quark. It is a good approximation for the b-quark. However, as our numerical analysis shows, such an approximation does not work well in the case of t-quark and higher terms in large mass expansion should be taken into account. We plan to calculate missing corrections from stop–chargino loops to the b-quark pole mass and to provide more terms in the expansion in the relation between the t-quark pole and $\overline{\mathrm{DR}}$ masses elsewhere.

This paper is organized as follows. In Sect. 2, we define the quantities we want to compute and make comments on the choice of our renormalization scheme. In Sect. 3, we

[1] The $\overline{\mathrm{DR}}$ scheme [4], which is similar to the $\overline{\mathrm{MS}}$ scheme, is usually used for calculation in supersymmetric models, because contrary to the $\overline{\mathrm{MS}}$ scheme it manifestly preserves supersymmetry, at least for not very high loop orders [5]. The details of the renormalization scheme used will be given below

describe our model, the supersymmetric QCD (a subset of MSSM, relevant to the calculation of α_s^n corrections), and present in detail the renormalization procedure. Section 4 contains our results and a numerical analysis of the effect produced by our correction on the supersymmetric particle spectra. The latter were computed with the help of the SoftSUSY program [6] under universal boundary conditions at the GUT scale. Finally, Sect. 5 contains our conclusions.

2 Pole mass and choice of renormalization scheme

First, we would like to note that there are several quark mass definitions. This is mainly due to the fact that quarks were never observed as free particles. Therefore, the definition of their masses relies heavily on theoretical constructions. Different definitions exist referring to different renormalization schemes used in quantum field theories. We will mention only a few most popular ones: $\overline{\text{MS}}$ mass $\overline{m}$, pole mass M and $\overline{\text{DR}}$ mass m. In the non-supersymmetric QCD the relations between these three masses are known (to two loops between $\overline{\text{DR}}$ and pole masses [7] and to two [8,9] and to three [10,11] loops between $\overline{\text{MS}}$ and pole masses) and in the supersymmetric QCD case there exists only one-loop relation between $\overline{\text{DR}}$ and pole masses of heavy quarks [12–16]. In this work, we will establish the two-loop relation between $\overline{\text{DR}}$ and pole masses in supersymmetric QCD.

In the $\overline{\text{DR}}$ scheme, particle masses and couplings depend explicitly on the renormalization scale $\bar{\mu}$ which is taken in applications equal to the characteristic scale of a studied process. The renormalization scale dependence of these quantities could be conveniently described by renormalization group equations. They are known very well for MSSM and supersymmetric QCD. It should be noted that the $\overline{\text{DR}}$ mass is a short distance quantity. That is, it is sensitive only to short distance effects. To describe processes with the characteristic scale of the order of the quark mass itself, a different on-shell mass definition is used and here the pole mass of a particle comes into play.

The particle pole mass is defined by the singularity of the corresponding two-point function. As explicit perturbative calculations showed, the pole mass of a quark is an infrared finite and gauge invariant quantity. For this reason it is considered as a physically meaningful quantity [17]. We will restrict ourselves to perturbation theory only and will not analyze the exact nature of the two-point function singularity which may involve some non-perturbative dynamics. In the present paper, we employ the definition of the pole quark mass, where it equals the value of p at which the inverse quark propagator turns to zero.

The pole mass M of a quark is defined as a formal solution for $\hat{p}$ (in the Minkowski metric) at which the reciprocal of the connected full propagator equals zero:

$$\hat{p} - m - \Sigma(\hat{p}, m) = 0, \qquad (1)$$

Table 1. Two groups of mass scales

	m_{soft}	m_{hard}
Case of b-quark:	m_b	m_t, $m_{\tilde{q}}$, $m_{\tilde{g}}$
Case of t-quark:	m_t	$m_{\tilde{q}}$, $m_{\tilde{g}}$

where $\Sigma(\hat{p}, m) = m\Sigma_1(p^2, m) + (\hat{p} - m)\Sigma_2(p^2, m)$ is the one-particle-irreducible two-point function, m may stand for the bare or renormalized mass, m_{bare} or m_{ren}, depending on the prescription used in evaluating Σ. The solution to (1) is sought order by order in perturbation theory. To two loops we have

$$\begin{aligned} M &= m + \Sigma^{(1)}(m, m) + \Sigma^{(2)}(m, m) \\ &\quad + \Sigma^{(1)}(m, m)\ \Sigma^{(1)'}(m, m) + \mathcal{O}(\Sigma^{(3)}), \end{aligned} \qquad (2)$$

where $\Sigma^{(L)}$ is the L-loop contribution to Σ, and the prime denotes the derivative with respect to the first argument. In what follows, we will be interested in the relation between the pole quark mass M and the running $\overline{\text{DR}}$ mass m computed in MSSM up to the $\mathcal{O}(\alpha_s^2)$ order. Technically, to solve this problem, we need to evaluate ≈ 200 two-loop propagator type diagrams involving many different mass scales (see Fig. 1). In general, this is quite a complex problem to be solved exactly. However, in our case all mass scales can be divided into two groups denoted by m_{soft} and m_{hard}, such that scales from m_{soft} are much less than each of the scales from the m_{hard} group; see Table 1. Here $m_{\tilde{g}}$ is the gluino mass, and $m_{\tilde{q}}$ stands for different squark masses. We also explicitly denoted which mass scales belong to soft or hard groups in the cases of b- and t-quarks.

Given this mass hierarchy, one can employ the large mass expansion procedure to reduce the evaluation of multiscale two-loop integrals to the calculation of single scale on-shell two-loop integrals, two-loop tadpole integrals with two scales and products of one-loop on-shell integrals and one-loop tadpole diagrams. To compute two-loop single scale on-shell integrals, we made use of the ONSHELL2 package [18]. For the evaluation of two-loop tadpole diagrams with two scales, the recurrence relations of [19] were used. Calculation of one-loop self-energies, including their derivatives with respect to momentum, is an easy task and we will not make further comments on it.

Now let us make some comments on the choice of the renormalization scheme. Here we use the same renormalization prescription as in [7]. To be more specific, we use regularization by dimensional reduction, a modification of the conventional dimensional regularization, originally proposed in [4]. In this procedure, the vector and spinor algebras in the numerator of Feynman diagrams are four-dimensional, which is the requirement imposed by supersymmetry. However, to make sense of divergent momentum integrals we need to regularize them by non-integer space-time dimension $d = 4 - 2\varepsilon$. For quantum corrections not to break gauge invariance, we need a cancellation of squared momenta in both the numerator and the denominator of Feynman integrals. Thus the momenta

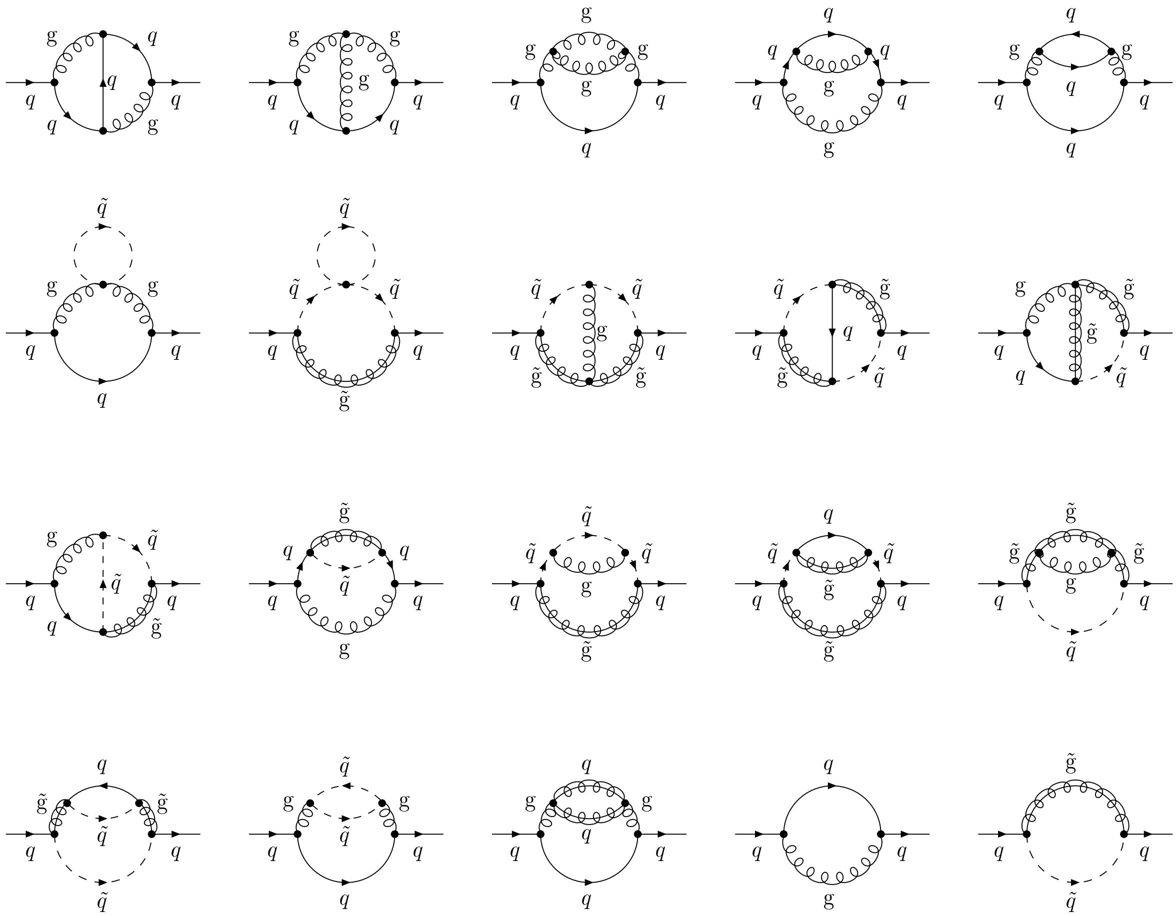

Fig. 1. Feynman diagrams for the $\mathcal{O}(\alpha_s^2)$ contribution to quark self-energies (last two diagrams at the one-loop level of the order $\mathcal{O}(\alpha_s)$). The diagrams in the first line are the pure QCD contribution

should form a d-dimensional subspace in four dimensions. As the momenta become d-dimensional, four-vectors naturally split into true d-vectors and so-called ε-scalars, obtained in the process of dimensional reduction of the original four-dimensional Lagrangian. The ε-scalars are nothing but matter fields. Their appearance is the only difference from the conventional dimensional regularization.

We would like to note that, in general, renormalization of ε-scalars and their interactions is not identical to that of vectors, so the original four-covariance may be spoiled by quantum corrections. Moreover, quantum corrections may also generate a mass for ε-scalars. There is arbitrariness in choosing the renormalization scheme for this mass [20]. A consistent way is to choose the finite ε-scalar mass counterterm, so that the pole (and renormalized) mass of the ε-scalars equals zero. The ε-scalar field renormalization is left minimal. Other renormalizations are done minimally, that is by subtracting only poles in ε.

3 Definition of the model and renormalization procedure

For our purposes here we do not need the complete MSSM, but only the part that is the supersymmetric extension of QCD (SUSY QCD). In the superfield formulation the SUSY QCD Lagrangian consists of two parts: the rigid supersymmetric part and the part containing soft supersymmetry breaking terms. We have

$$\begin{aligned}\mathcal{L}_{\text{rigid}} &= \int \mathrm{d}^2\theta\, \frac{1}{2}\, \mathrm{Tr} W^\alpha W_\alpha + \int \mathrm{d}^2\bar{\theta}\, \frac{1}{2}\, \mathrm{Tr}\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} \\ &+ \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, \Big[\bar{Q}\mathrm{e}^{2g_3 T^a V^a} Q + \bar{U}\mathrm{e}^{-2g_3 (T^a)^* V^a} U \\ &+ \bar{D}\mathrm{e}^{-2g_3 (T^a)^* V^a} D\Big] + \int \mathrm{d}^2\theta\, \mathcal{W} + \int \mathrm{d}^2\bar{\theta}\, \bar{\mathcal{W}}, \quad (3)\end{aligned}$$

where $Q = Q_\mathrm{L}$, $U = U_\mathrm{R}$, $D = D_\mathrm{R}$, $T^a = \lambda^a/2$ ($a = 1, \cdots, 8$) with λ^a being the Gell-Mann matrices[2], and the gauge field strength tensors are

$$\begin{aligned} W_\alpha &= -\frac{1}{8g_3} \bar{D}^2 \mathrm{e}^{-2g_3 T^a V^a} D_\alpha \mathrm{e}^{2g_3 T^a V^a}, \\ \bar{W}_{\dot{\alpha}} &= -\frac{1}{8g_3} D^2 \mathrm{e}^{2g_3 T^a V^a} \bar{D}_{\dot{\alpha}} \mathrm{e}^{-2g_3 T^a V^a}. \end{aligned}$$

The superpotential of the ordinary SUSY QCD contains only quadratic mass terms for chiral (antichiral) superfields. Considering SUSY QCD as a part of MSSM,

[2] Note that U and D are color antitriplets and the anticolor generator is $-(T^a)^*$

where all masses of particles are generated through the Higgs mechanism, the superpotential takes the following form:

$$\mathcal{W} = \epsilon^{ij}\left[y_d D H_1^i Q^j + y_u U H_2^j Q^i + \mu H_1^j H_2^i\right], \qquad (4)$$

where the y_d and y_u are the Yukawa coupling constants carrying the generation indices which have been suppressed (as well as group indices). In this work we will need the Higgs fields only to generate masses of our particles and will not be interested in the detailed structure of the MSSM Higgs sector.

To perform the supersymmetry breaking that satisfies the requirement of "softness", we introduce a gluino mass term, soft masses of scalar superpartners of quarks and soft trilinear couplings with Higgs fields, which may be written in terms of $N=1$ superfields, provided one introduces external spurion superfields [21]

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &\\ -&\frac{1}{2}\int \mathrm{d}^2\theta\, 2M_3\theta^2 \mathrm{Tr}W^\alpha W_\alpha - \frac{1}{2}\int \mathrm{d}^2\bar\theta\, 2\bar M_3\bar\theta^2\mathrm{Tr}\bar W_{\dot\alpha}\bar W^{\dot\alpha}\\ -&\int \mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta\left[M_{\tilde q}^2\bar Q \mathrm{e}^{2g_3T^aV^a}Q + M_{\tilde u}^2\bar U\mathrm{e}^{-2g_3(T^a)^*V^a}U\right.\\ &\qquad\left. + \quad M_{\tilde d}^2\bar D\mathrm{e}^{-2g_3(T^a)^*V^a}D\right]\theta^2\bar\theta^2 \qquad (5)\\ -&\int \mathrm{d}^2\theta\,\epsilon^{ij}\left[y_dA_dDH_1^iQ^j + y_uA_uUH_2^jQ^i\right]\theta^2 + \text{h.c.}\end{aligned}$$

The chiral superfields have the following component fields representations:

$$\begin{aligned}Q &= \tilde q_{\mathrm L} - \mathrm{i}\,\theta\sigma^\mu\bar\theta\partial_\mu\tilde q_{\mathrm L} - \frac14\theta\theta\bar\theta\bar\theta\partial^\mu\partial_\mu\tilde q_{\mathrm L} + \sqrt2\theta q_{\mathrm L}\\ &\quad - \frac{\mathrm i}{\sqrt2}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu q_{\mathrm L} + \theta\theta F_q, \qquad (6)\\ U &= \bar{\tilde u}_{\mathrm R} - \mathrm{i}\,\theta\sigma^\mu\bar\theta\partial_\mu\bar{\tilde u}_{\mathrm R} - \frac14\theta\theta\bar\theta\bar\theta\partial^\mu\partial_\mu\bar{\tilde u}_{\mathrm R} + \sqrt2\theta u_{\mathrm R}\\ &\quad - \frac{\mathrm i}{\sqrt2}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu u_{\mathrm R} + \theta\theta F_u, \qquad (7)\\ D &= \bar{\tilde d}_{\mathrm R} - \mathrm{i}\,\theta\sigma^\mu\bar\theta\partial_\mu\bar{\tilde d}_{\mathrm R} - \frac14\theta\theta\bar\theta\bar\theta\partial^\mu\partial_\mu\bar{\tilde d}_{\mathrm R} + \sqrt2\theta d_{\mathrm R}\\ &\quad - \frac{\mathrm i}{\sqrt2}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu d_{\mathrm R} + \theta\theta F_d, \qquad (8)\end{aligned}$$

and for the vector superfields in the Wess–Zumino gauge we have

$$V^a = \theta\sigma^\mu\bar\theta G_\mu^a + \theta\theta\bar\theta\bar{\tilde g}^a + \bar\theta\bar\theta\theta\tilde g^a + \frac12\theta\theta\bar\theta\bar\theta D^a. \qquad (9)$$

Then, rewriting everything in the component fields, the Lagrangian of SUSY QCD takes the form ($q_{\mathrm R} = (u_{\mathrm R}, d_{\mathrm R})$)

$$\begin{aligned}\mathcal{L} = &-\frac14 G_{\mu\nu}^a G^{a\,\mu\nu} + \mathrm{i}\bar{\tilde g}^a\bar\sigma^\mu D_\mu\tilde g^a + \frac12 D^aD^a + \mathrm{i}\bar q_{\mathrm R}\bar\sigma^\mu D_\mu q_{\mathrm R}\\ &+ \mathrm{i}\bar q_{\mathrm L}\bar\sigma^\mu D_\mu q_{\mathrm L} + (D^\mu\tilde q_{\mathrm R})^\dagger D_\mu\tilde q_{\mathrm R} + (D^\mu\tilde q_{\mathrm L})^\dagger D_\mu\tilde q_{\mathrm L}\\ &- g_3\bar{\tilde q}_{\mathrm R}T^aD^a\tilde q_{\mathrm R} + g_3\bar{\tilde q}_{\mathrm L}T^aD^a\tilde q_{\mathrm L} + \sqrt2 g_3 q_{\mathrm R}T^a\tilde g^a\tilde q_{\mathrm R}\\ &- \sqrt2 g_3\bar q_{\mathrm L}T^a\bar{\tilde g}^a\tilde q_{\mathrm L} + \sqrt2 g_3\bar{\tilde q}_{\mathrm R}T^a\bar{\tilde g}^a\bar q_{\mathrm R} - \sqrt2 g_3\bar{\tilde q}_{\mathrm L}T^a\tilde g^a q_{\mathrm L}\\ &+ \frac12 D^iD^i + \frac12 D'D' + g_2\bar{\tilde q}_{\mathrm L}\frac{\sigma^i}{2}D^i\tilde q_{\mathrm L} + g_2\bar h_j\frac{\sigma^i}{2}D^ih_j\\ &+ g_1\bar h_j^0\frac{Y_{h_j}}{2}D'h_j^0 + g_1\bar{\tilde q}_{\mathrm L}\frac{Y_{\tilde q_{\mathrm L}}}{2}D'\tilde q_{\mathrm L} - g_1\bar{\tilde q}_{\mathrm R}\frac{Y_{\tilde q_{\mathrm R}}}{2}D'\tilde q_{\mathrm R}\\ &+ \bar F_{h_1^0}F_{h_1^0} + \bar F_{h_2^0}F_{h_2^0} + \bar F_{q_{\mathrm L}}F_{q_{\mathrm L}} + \bar F_{q_{\mathrm R}}F_{q_{\mathrm R}}\\ &- \Big[\mu h_1^0F_{h_2^0} + \mu F_{h_1^0}h_2^0 - y_d\bar{\tilde d}_{\mathrm R}F_{h_1^0}\tilde d_{\mathrm L} - y_u\bar{\tilde u}_{\mathrm R}F_{h_2^0}\tilde u_{\mathrm L}\\ &- y_d(F_{d_{\mathrm R}}h_1^0\tilde d_{\mathrm L} + \bar{\tilde d}_{\mathrm R}h_1^0F_{d_{\mathrm L}}) - y_u(F_{u_{\mathrm R}}h_2^0\tilde u_{\mathrm L} + \bar{\tilde u}_{\mathrm R}h_2^0F_{u_{\mathrm L}})\\ &+ y_d d_{\mathrm R}h_1^0 d_{\mathrm L} + y_u u_{\mathrm R}h_2^0u_{\mathrm L} + y_dA_d\tilde d_{\mathrm R}h_1^0\tilde d_{\mathrm L}\\ &+ y_uA_u\tilde u_{\mathrm R}h_2^0\tilde u_{\mathrm L} + \text{h.c.}\Big] - \frac{M_3}{2}\tilde g^a\tilde g^a - \frac{M_3}{2}\bar{\tilde g}^a\bar{\tilde g}^a\\ &- M_{\tilde q}^2\bar{\tilde q}_{\mathrm L}\tilde q_{\mathrm L} - M_{\tilde u}^2\bar{\tilde u}_{\mathrm R}\tilde u_{\mathrm R} - M_{\tilde d}^2\bar{\tilde d}_{\mathrm R}\tilde d_{\mathrm R},\end{aligned}$$

where the gauge covariant derivative and field strength are defined by

$$\begin{aligned}D_\mu &= \partial_\mu + \mathrm{i}g_3T^aG_\mu^a,\\ G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_3f^{abc}G_\mu^bG_\nu^c.\end{aligned}$$

Here we also added the auxiliary $SU(2)\times U(1)$ fields D^i and D', which are necessary to describe correctly the mass generation mechanism. The σ^i denote Pauli matrices and Y_q is the hypercharge of a particle q. Eliminating auxiliary fields F_q, $\bar F_q$, D^a, D^i, and D', and introducing a four-component spinor notation similar to [1,22], one gets

$$\begin{aligned}\mathcal{L} = &-\frac14 G_{\mu\nu}^aG^{a\,\mu\nu} + \frac{\mathrm i}{2}\bar{\tilde g}^a\gamma^\mu D_\mu\tilde g^a + \mathrm{i}\bar q\gamma^\mu D_\mu q\\ &+ (D^\mu\tilde q_{\mathrm R})^\dagger D_\mu\tilde q_{\mathrm R} + (D^\mu\tilde q_{\mathrm L})^\dagger D_\mu\tilde q_{\mathrm L}\\ &+ \sqrt2 g_3\bar qP_{\mathrm L}T^a\tilde g^a\tilde q_{\mathrm R} - \sqrt2 g_3\bar qP_{\mathrm R}T^a\tilde g^a\tilde q_{\mathrm L}\\ &+ \sqrt2 g_3\bar{\tilde q}_{\mathrm R}T^a\bar{\tilde g}^aP_{\mathrm R}q - \sqrt2 g_3\bar{\tilde q}_{\mathrm L}T^a\bar{\tilde g}^aP_{\mathrm L}q\\ &- \frac{g_3^2}{2}(\bar{\tilde q}_{\mathrm L}T^a\tilde q_{\mathrm L} - \bar{\tilde q}_{\mathrm R}T^a\tilde q_{\mathrm R})(\bar{\tilde q}_{\mathrm L}T^a\tilde q_{\mathrm L} - \bar{\tilde q}_{\mathrm R}T^a\tilde q_{\mathrm R})\\ &- \frac{M_3}{2}\bar{\tilde g}^a\tilde g^a + \mathcal{L}_M, \qquad (10)\\ \mathcal{L}_M = &-M_{\tilde q}^2\bar{\tilde q}_{\mathrm L}\tilde q_{\mathrm L} - M_{\tilde u}^2\bar{\tilde u}_{\mathrm R}\tilde u_{\mathrm R} - M_{\tilde d}^2\bar{\tilde d}_{\mathrm R}\tilde d_{\mathrm R} - y_dh_1^0\bar dd\\ &- y_uh_2^0\bar uu - \left(\bar h_1^0h_1^0 - \bar h_2^0h_2^0\right)\\ &\times\left(\frac{g_2^2}{2}I_{\tilde q_{\mathrm L}}^3\bar{\tilde q}_{\mathrm L}\tilde q_{\mathrm L} - \frac{g_1^2}{4}Y_{\tilde q_{\mathrm L}}\bar{\tilde q}_{\mathrm L}\tilde q_{\mathrm L} + \frac{g_1^2}{4}Y_{\tilde q_{\mathrm R}}\bar{\tilde q}_{\mathrm R}\tilde q_{\mathrm R}\right)\\ &- y_d^2\left(h_1^0\tilde d_{\mathrm L}\right)^\dagger h_1^0\tilde d_{\mathrm L} - y_d^2\bar h_1^0h_1^0\bar{\tilde d}_{\mathrm R}\tilde d_{\mathrm R}\\ &- y_u^2\left(h_2^0\tilde u_{\mathrm L}\right)^\dagger h_2^0\tilde u_{\mathrm L} - y_u^2\bar h_2^0h_2^0\bar{\tilde u}_{\mathrm R}\tilde u_{\mathrm R}\\ &- \Big[y_dh_1^0A_d\bar{\tilde d}_{\mathrm R}\tilde d_{\mathrm L} + y_uh_2^0A_u\bar{\tilde u}_{\mathrm R}\tilde u_{\mathrm L} - y_d\mu h_2^0\bar{\tilde d}_{\mathrm R}\tilde d_{\mathrm L}\\ &- y_u\mu h_1^0\bar{\tilde u}_{\mathrm R}\tilde u_{\mathrm L} + \text{h.c}\Big], \qquad (11)\end{aligned}$$

where $P_{\mathrm L} = (1-\gamma_5)/2$ and $P_{\mathrm R} = (1+\gamma_5)/2$ are the chiral and antichiral projectors.

Performing electroweak symmetry breaking and taking into account that

$$m_d = y_dv_1, \qquad m_u = y_uv_2, \qquad \tan\beta = \frac{v_2}{v_1},$$

$$m_Z^2 = \frac{1}{2}(g_1^2+g_2^2)(v_1^2+v_2^2) = \frac{1}{2}\frac{g_1^2}{\sin^2\theta_{\rm W}}(v_1^2+v_2^2),$$
$$\cos 2\beta = \frac{v_1^2-v_2^2}{v_1^2+v_2^2}, \qquad e_q = \frac{Y_q}{2}+I_q^3,$$

it is convenient to rewrite $\mathcal{L}_M$ from (11) in the following form [23]:

$$\mathcal{L}_M = -m_q\bar{q}q - m_{\tilde{q}_{\rm L}}^2\bar{\tilde{q}}_{\rm L}\tilde{q}_{\rm L} - m_{\tilde{u}_{\rm R}}^2\bar{\tilde{u}}_{\rm R}\tilde{u}_{\rm R} - m_{\tilde{d}_{\rm R}}^2\bar{\tilde{d}}_{\rm R}\tilde{d}_{\rm R} - a_q m_q\bar{\tilde{q}}_{\rm L}\tilde{q}_{\rm R} - a_q m_q\bar{\tilde{q}}_{\rm R}\tilde{q}_{\rm L}, \qquad (12)$$

with

$$\begin{aligned} m_{\tilde{q}_{\rm L}}^2 &= M_{\tilde{q}}^2 + m_Z^2\cos 2\beta\,(I_{\tilde{q}_{\rm L}}^3 - e_q\sin^2\theta_{\rm W}) + m_q^2\,,\\ m_{\tilde{u}_{\rm R}}^2 &= M_{\tilde{u}}^2 + e_u\,m_Z^2\cos 2\beta\,\sin^2\theta_{\rm W} + m_u^2\,,\\ a_u &= A_u - \mu\,\cot\beta,\\ m_{\tilde{d}_{\rm R}}^2 &= M_{\tilde{d}}^2 + e_d\,m_Z^2\cos 2\beta\,\sin^2\theta_{\rm W} + m_d^2\,,\\ a_d &= A_d - \mu\,\tan\beta, \end{aligned}$$

where e_q and I_q^3 are the electric charge and the third component of the weak isospin of a particle q, m_q is the mass of the partner quark, and μ is a Higgs mass parameter. The $M_{\tilde{q}}$, $M_{\tilde{u}}$, and $M_{\tilde{d}}$ are soft SUSY breaking masses, A_u and A_d are trilinear couplings as in (5). The family indices have been suppressed.

Now we must diagonalize the mass matrix $\mathcal{M}_{\tilde{q}}^2$ of the squarks to determine the physical mass eigenstates:

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} m_{\tilde{q}_{\rm L}}^2 & a_q m_q \\ a_q m_q & m_{\tilde{q}_{\rm R}}^2 \end{pmatrix} = (\mathcal{R}^{\tilde{q}})^\dagger \begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix}\mathcal{R}^{\tilde{q}}\,. \qquad (13)$$

According to (13) $\mathcal{M}_{\tilde{q}}^2$ is diagonalized by a unitary matrix $\mathcal{R}^{\tilde{q}}$. Assuming that CP violating phases occur only in the CKM matrix, we choose $\mathcal{R}^{\tilde{q}}$ to be real. The weak eigenstates $\tilde{q}_{\rm L}$ and $\tilde{q}_{\rm R}$ are thus related to their mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ by

$$\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix} = \mathcal{R}^{\tilde{q}}\begin{pmatrix}\tilde{q}_{\rm L}\\ \tilde{q}_{\rm R}\end{pmatrix}, \qquad \mathcal{R}^{\tilde{q}} = \begin{pmatrix} \cos\theta_{\tilde{q}} & \sin\theta_{\tilde{q}} \\ -\sin\theta_{\tilde{q}} & \cos\theta_{\tilde{q}}\end{pmatrix}, \qquad (14)$$

with $\theta_{\tilde{q}}$ being the squark mixing angle. The mass eigenvalues are given by

$$m_{\tilde{q}_{1,2}}^2 = \frac{1}{2}\left(m_{\tilde{q}_{\rm L}}^2 + m_{\tilde{q}_{\rm R}}^2 \mp \sqrt{(m_{\tilde{q}_{\rm L}}^2 - m_{\tilde{q}_{\rm R}}^2)^2 + 4\,a_q^2 m_q^2}\right). \qquad (15)$$

By convention, we choose $\tilde{q}_1$ to be the lightest mass eigenstate. Notice that $m_{\tilde{q}_1} \le m_{\tilde{q}_{\rm L,R}} \le m_{\tilde{q}_2}$.

For the mixing angle $\theta_{\tilde{q}}$ we require $0 \le \theta_{\tilde{q}} < \pi$. Then one can write

$$\cos\theta_{\tilde{q}} = \frac{-a_q m_q}{\sqrt{(m_{\tilde{q}_{\rm L}}^2 - m_{\tilde{q}_1}^2)^2 + a_q^2 m_q^2}},$$
$$\sin\theta_{\tilde{q}} = \frac{m_{\tilde{q}_{\rm L}}^2 - m_{\tilde{q}_1}^2}{\sqrt{(m_{\tilde{q}_{\rm L}}^2 - m_{\tilde{q}_1}^2)^2 + a_q^2 m_q^2}}. \qquad (16)$$

Moreover, $|\cos\theta_{\tilde{q}}| > \frac{1}{\sqrt{2}}$ if $m_{\tilde{q}_{\rm L}} < m_{\tilde{q}_{\rm R}}$ and $|\cos\theta_{\tilde{q}}| < \frac{1}{\sqrt{2}}$ if $m_{\tilde{q}_{\rm R}} < m_{\tilde{q}_{\rm L}}$.

Making the transition in (10) from $\tilde{q}_{\rm R}$ and $\tilde{q}_{\rm L}$ to $\tilde{q}_1$ and $\tilde{q}_2$ with the help of the matrix from (14) we get for the quark–gluino–squark trilinear and four-squark vertices complicated expressions due to squark mixing which we do not present here. To produce input amplitudes of the diagrams needed in our calculations we made use of the Mathematica package FeynArts [24], which has SUSY QCD as part of the MSSM model [25]. The Feynman diagrams contributing to quark self-energies in the one- and two-loop order are shown in Fig. 1.

Note again that doing these calculations we work in the minimal subtraction scheme $\overline{\rm MS}$ adopted to supersymmetry – dimensional reduction with minimal subtraction $\overline{\rm DR}$. To obtain a relation between pole and $\overline{\rm DR}$ quark masses, we need to know $\overline{\rm DR}$ the renormalization constants (at one-loop order they coincide with those calculated using the $\overline{\rm MS}$ prescription) for quark, gluino, squark, ε-scalar masses, gauge and Yukawa charges[3] as well as renormalization of the squark mixing angle. We will describe the evaluation of each of these quantities in the next subsections.

3.1 Renormalization of the QCD sector

First we have to renormalize the charge and the masses of quarks and fields. The relations between bare and renormalized parameters are given by

$$\alpha_{\rm s,0} = \bar{\mu}^{2\,\varepsilon}Z_{\alpha_{\rm s}}\alpha_{\rm s}\,, \qquad (17)$$
$$m_{q,0} = Z_{m_q}m_q\,, \qquad (18)$$
$$G_0 = Z_G^{1/2}G\,, \qquad (19)$$
$$q_0 = Z_q^{1/2}q\,, \qquad (20)$$

where $\bar{\mu}$ is a renormalization scale.

The charge renormalization constant can be obtained from the renormalization constants of the fields and vertex with the use of the following relation:

$$Z_{\alpha_{\rm s}} = Z_{qGq}^2\,Z_q^{-2}\,Z_G^{-1}\,, \qquad (21)$$

where Z_q and Z_G correspond to renormalization of quark and gluon fields and Z_{qGq} renormalizes the quark–gluon interaction vertex.

One can also renormalize the gauge parameter[4] with the help of the above-introduced constant Z_G

$$\xi_0 = Z_G\xi\,. \qquad (22)$$

However, in the gauge invariant quantities (like, e.g., the pole mass) the gauge parameter drops out already from

[3] By Yukawa charge we mean the Yukawa coupling of the ε-scalar to the fermions

[4] The gauge parameter ξ is defined in such a way that $\xi = 0$ and $\xi = 1$ correspond to the Landau and Feynman gauge, respectively

the bare expression and the renormalization (22) is not needed.

Our results for the renormalization constants in the $\overline{\mathrm{DR}}$ scheme read

$$Z_{m_q} = 1 - C_F \frac{\alpha_s}{2\pi}\frac{1}{\varepsilon} - C_F\left(\frac{\alpha_s}{4\pi}\right)^2 \left((6 - 3C_A - 2C_F)\frac{1}{\varepsilon^2} + (-6 + 3C_A - 2C_F)\frac{1}{\varepsilon}\right), \quad (23)$$

$$Z_q = 1 - \frac{\alpha_s}{4\pi} C_F (\xi + 1)\frac{1}{\varepsilon}, \quad (24)$$

$$Z_G = 1 - \frac{\alpha_s}{4\pi}\left(6 - \frac{3-\xi}{2}C_A\right)\frac{1}{\varepsilon}, \quad (25)$$

$$Z_{qGq} = 1 - \frac{\alpha_s}{4\pi}\left((1+\xi)C_F + \frac{3+\xi}{4}C_A\right)\frac{1}{\varepsilon}, \quad (26)$$

$$Z_{\alpha_s} = 1 - \frac{\alpha_s}{4\pi}(3C_A - 6)\frac{1}{\varepsilon}. \quad (27)$$

For $SU(N_c)$ we have $C_F = (N_c^2 - 1)/(2N_c)$ and $C_A = N_c$. Here we explicitly put the number of quark flavors to six. Equations (23)–(27) coincide with the corresponding renormalization constants from [26].

There are several possibilities to check the correctness of our results. The coupling renormalization constant (27) can be compared with that obtained from the one-loop MSSM beta function for the $SU(3)$ group. We have also checked that in the case of ordinary QCD we recover the renormalization constants given in [7]. The two-loop $\overline{\mathrm{DR}}$ quark mass renormalization constant (23) has been obtained from (2) and contains poles in ε remaining after all other renormalizations were done. Here we mean that one should perform one-loop renormalizations of the strong coupling constant and particle masses. A subtle point is the renormalization of the Yukawa interaction of ε-scalars and quarks. In SUSY QCD as discussed here, one does not need to introduce a separate coupling constant Y for this interaction, as supersymmetry ensures that it is renormalized in the same way as a strong coupling constant[5]. However, to make our renormalization procedure as general as possible (and also applicable in the case of non-supersymmetric QCD for checks), we introduce an additional independent constant Y and renormalize it separately. So, here we follow the renormalization procedure described in [7]. Renormalizing a physical quantity like a pole mass, we ignore field renormalization constants, which is a justified procedure, provided we are evaluating the contribution of counterterms for the whole sum of diagrams. The $1/\varepsilon^2$ poles in Z_{m_q} can be checked with the use of the renormalization group technique. Indeed, let us write

$$m_0 = m\left(1 + \frac{\alpha_s}{4\pi}\frac{Z^{(1,1)}}{\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{Z^{(2,1)}}{\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{Z^{(2,2)}}{\varepsilon^2}\right)$$

[5] See the section about the ε-scalar sector renormalization for details

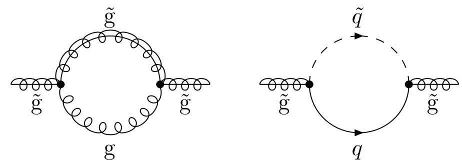

Fig. 2. One-loop diagrams contributing to gluino mass counterterm

$$= m\left(1 + \frac{z^{(1)}}{\varepsilon} + \frac{z^{(2)}}{\varepsilon^2} + \ldots\right), \quad (28)$$

where

$$z^{(n)} = \sum_{l=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^l Z^{(l,n)}. \quad (29)$$

Now using the renormalization group equation[6], we have

$$0 = \bar{\mu}^2 \frac{\mathrm{d}}{\mathrm{d}\bar{\mu}^2} m_0,$$

and we obtain

$$\gamma = \frac{1}{2} g \frac{\partial}{\partial g} z^{(1)},$$

$$\left(\gamma + \beta_g \frac{\partial}{\partial g} + \sum_i \gamma_i m_i \frac{\partial}{\partial m_i}\right) z^{(n)} = \frac{1}{2} g \frac{\partial}{\partial g} z^{(n+1)} \quad (30)$$

(g is related to α_s via $g^2/4\pi = \alpha_s$). The formulae presented are general and are also applicable in a theory with spontaneously broken symmetry [27]. In the case of SUSY QCD, from the previous formula it follows that

$$2Z^{(2,2)} = \left(Z^{(1,1)}\right)^2 + 2\beta_g^{(1)} Z^{(1,1)}. \quad (31)$$

Substituting into this relation the expressions for $Z^{(1,1)} = -2C_F$ and $\beta_g^{(1)} = -\frac{1}{2}(3C_A - 6)$, we get the same result for the $1/\varepsilon^2$ poles as in (23).

3.2 Gluino mass renormalization

The renormalization of the gluino mass is performed in a way similar to quark mass renormalization. The relevant diagrams with gluino–gluon and quark–squark loops are shown in Fig. 2. Explicit calculations give the following results for the gluino mass $m_{\tilde{g},0} = Z_{m_{\tilde{g}}} m_{\tilde{g}}$, for the gluino wave function renormalization constants $Z_{\tilde{g}}$ and for the gluino–gluon–gluino vertex renormalization constant $Z_{\tilde{g}G\tilde{q}}$

[6] In fact m_0 is not a renormalization group invariant in our renormalization scheme. However, to check the leading poles in ε we can treat it as an RG invariant, as different prescriptions used to renormalize the ε-scalar sector give different results only in the subleading order in ε. This point will be discussed in detail in the section on the ε-scalar sector renormalization

[28]:

$$Z_{m_{\tilde{g}}} = 1 + \frac{\alpha_s}{4\pi}(6 - 3C_A)\frac{1}{\varepsilon}, \quad (32)$$

$$Z_{\tilde{g}} = 1 - \frac{\alpha_s}{4\pi}(\xi C_A + 6)\frac{1}{\varepsilon}, \quad (33)$$

$$Z_{\tilde{g}G\tilde{g}} = 1 - \frac{\alpha_s}{4\pi}\left(6 + \frac{3+5\xi}{4}C_A\right)\frac{1}{\varepsilon}. \quad (34)$$

From (25), (33) and (34) one can extract the gauge coupling renormalization constant

$$Z_{\alpha_s} = Z_{\tilde{g}G\tilde{g}}^2 Z_{\tilde{g}}^{-2} Z_G^{-1}, \quad (35)$$

which, of course, coincides with (27). This serves as an additional check. The result for the gluino mass renormalization constant can also be verified with help of the known gluino mass beta function.

3.3 Squark sector renormalization

To discuss the renormalization of the squark sector let us start with the Lagrangian written in terms of physical states. The bare Lagrangian in terms of bare parameters and bare fields is given by[7]

$$\mathcal{L}_{\tilde{q}}^{\text{bare}} = \partial_\mu \bar{\tilde{q}}_i \partial_\mu \tilde{q}_i - m_{\tilde{q}_i}^2 \bar{\tilde{q}}_i \tilde{q}_i + \mathcal{L}_{q\tilde{g}\tilde{q}}^{\text{bare}}, \quad (36)$$

$$\begin{aligned}\mathcal{L}_{q\tilde{g}\tilde{q}}^{\text{bare}} &= -\sqrt{2}g_3\bar{q}\,(P_R\cos\theta_{\tilde{q}} - P_L\sin\theta_{\tilde{q}})\,T^a\tilde{g}^a\tilde{q}_1 \\ &+ \sqrt{2}g_3\bar{q}\,(P_R\sin\theta_{\tilde{q}} + P_L\cos\theta_{\tilde{q}})\,T^a\tilde{g}^a\tilde{q}_2 \\ &- \sqrt{2}g_3\bar{\tilde{q}}_1T^a\bar{\tilde{g}}^a\,(P_L\cos\theta_{\tilde{q}} - P_R\sin\theta_{\tilde{q}})\,q \\ &+ \sqrt{2}g_3\bar{\tilde{q}}_2T^a\bar{\tilde{g}}^a\,(P_L\sin\theta_{\tilde{q}} + P_R\cos\theta_{\tilde{q}})\,q. \quad (37)\end{aligned}$$

We can rewrite this Lagrangian in terms of renormalized physical quantities and counterterms:

$$\mathcal{L}_{\tilde{q}}^{\text{bare}} = \mathcal{L}_{\tilde{q}} + \delta\mathcal{L}_{\tilde{q}}, \quad (38)$$

$$\begin{aligned}\delta\mathcal{L}_{\tilde{q}} &= \delta Z_{\tilde{q}}(\partial_\mu\bar{\tilde{q}}_i\partial_\mu\tilde{q}_i - m_{\tilde{q}_i}^2\bar{\tilde{q}}_i\tilde{q}_i) - \delta m_{\tilde{q}_i}^2\bar{\tilde{q}}_i\tilde{q}_i \\ &- \delta\tilde{m}_{12}^2(\bar{\tilde{q}}_2\tilde{q}_1 + \bar{\tilde{q}}_1\tilde{q}_2) + \delta Z_{q\tilde{g}\tilde{q}}\mathcal{L}_{q\tilde{g}\tilde{q}}, \quad (39)\end{aligned}$$

where $\mathcal{L}_{\tilde{q}}$ and $\mathcal{L}_{q\tilde{g}\tilde{q}}$ have the same form as (36) and (37). It should be noticed that this set of counterterms is sufficient to cancel all rising divergencies. The $\delta\tilde{m}_{12}^2$ counterterm is used to render non-diagonal squark self-energies $\tilde{\Sigma}_{\tilde{q}_i\tilde{q}_j}$ ($i \neq j$) finite. In practical calculations, it is convenient to trade $\delta\tilde{m}_{12}^2$ for the counterterm for the squark mixing angle. Indeed, we can diagonalize the Lagrangian (38) with the following field redefinitions:

$$\begin{pmatrix}\tilde{q}_1'\\ \tilde{q}_2'\end{pmatrix} = \begin{pmatrix}1 & \delta\theta_{\tilde{q}}\\ -\delta\theta_{\tilde{q}} & 1\end{pmatrix}\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix},$$

$$\mathcal{R}^{\tilde{q}} = \begin{pmatrix}1 & \delta\theta_{\tilde{q}}\\ -\delta\theta_{\tilde{q}} & 1\end{pmatrix},$$

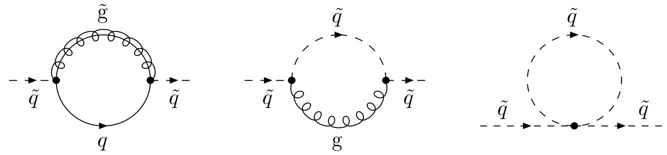

Fig. 3. One-loop diagrams contributing to the squark mass and squark mixing angle counterterm

where we have taken into account that $\delta\theta_{\tilde{q}} \sim \mathcal{O}(\alpha_s)$, $\cos(\delta\theta_{\tilde{q}}) = 1 + \mathcal{O}(\alpha_s^2)$ and $\sin(\delta\theta_{\tilde{q}}) = \delta\theta_{\tilde{q}} + \mathcal{O}(\alpha_s^3)$. Demanding that the new mass matrix of the squarks

$$\mathcal{R}^{\tilde{q}}\left[\mathcal{M}_{\tilde{q}}^2\right]^{(1)}\left(\mathcal{R}^{\tilde{q}}\right)^\dagger, \quad (40)$$

with

$$\left[\mathcal{M}_{\tilde{q}}^2\right]^{(1)} = \begin{pmatrix} m_{\tilde{q}_1}^2 + \delta Z_{\tilde{q}}m_{\tilde{q}_1}^2 + \delta m_{\tilde{q}_1}^2 & \delta\tilde{m}_{12}^2 \\ \delta\tilde{m}_{12}^2 & m_{\tilde{q}_2}^2 + \delta Z_{\tilde{q}}m_{\tilde{q}_2}^2 + \delta m_{\tilde{q}_2}^2\end{pmatrix},$$

must be diagonal and expanding (40) in the gauge coupling constant α_s, one gets

$$\delta\theta_{\tilde{q}} = \frac{\delta\tilde{m}_{12}^2}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} = \frac{(\tilde{\Sigma}_{\tilde{q}_1\tilde{q}_2}^{(1)})_{\text{div}}}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}, \quad (41)$$

where the subscript "div" stands for the $1/\varepsilon$ part of the expression. Rewriting (38) in terms of the new fields $\tilde{q}_i'$, we come to the following renormalization prescription for the squark sector:

$$m_{\tilde{q},0}^2 = m_{\tilde{q}_i}^2 + \delta m_{\tilde{q}_i}^2 = Z_{m_{\tilde{q}_i}^2} m_{\tilde{q}_i}^2, \quad (42)$$

$$\tilde{q}_{i,0} = \left(1 + \frac{1}{2}Z_{\tilde{q}}\right)\tilde{q}_i = Z_{\tilde{q}}^{1/2}\tilde{q}_i, \quad (43)$$

$$\theta_{\tilde{q},0} = \theta_{\tilde{q}} + \delta\theta_{\tilde{q}} = Z_{\theta_{\tilde{q}}}\theta_{\tilde{q}}. \quad (44)$$

Explicit calculation of squark self-energies gives (see the diagrams in Fig. 3) the following expression for the renormalization of the squark mixing angle:

$$\delta\theta_{\tilde{q}} = C_F\frac{\alpha_s}{4\pi}\left(-\frac{\sin(4\theta_{\tilde{q}})}{2} + \frac{4m_q m_{\tilde{g}}\cos(2\theta_{\tilde{q}})}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}\right)\frac{1}{\varepsilon}. \quad (45)$$

Note that the renormalization (45) of the mixing angle is minimal. It is the correct choice, if one wants to have the final result expressed in terms of the running $\overline{\text{DR}}$ sparticle masses.

Summarizing the result of our calculation, the renormalization constants for the squark sector read

$$Z_{\tilde{q}} = 1 - C_F\frac{\alpha_s}{4\pi}(\xi - 1)\frac{1}{\varepsilon}, \quad (46)$$

$$\begin{aligned}Z_{m_{\tilde{q}_1}^2} &= 1 + \frac{C_F}{m_{\tilde{q}_1}^2}\frac{\alpha_s}{4\pi}\Big(\left(m_{\tilde{q}_2}^2 - m_{\tilde{q}_1}^2\right)\sin^2 2\theta_{\tilde{q}} \\ &+ 4\left(m_q m_{\tilde{g}}\sin 2\theta_{\tilde{q}} - m_q^2 - m_{\tilde{g}}^2\right)\Big)\frac{1}{\varepsilon}, \quad (47)\end{aligned}$$

[7] We do not include the bare Lagrangian for a four-squark interaction, because the counterterms associated with the four-squark vertices will give a contribution of the $\mathcal{O}(\alpha_s^3)$ order

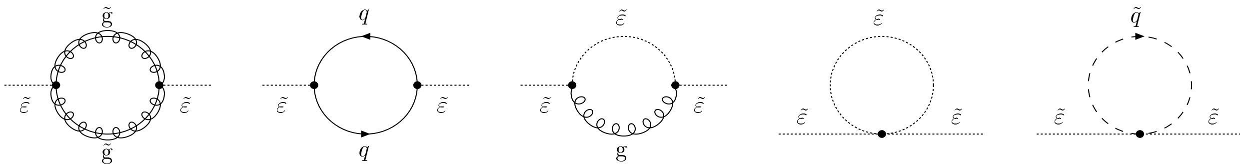

Fig. 4. One-loop diagrams contributing to the ε-scalar wave function and mass counterterms

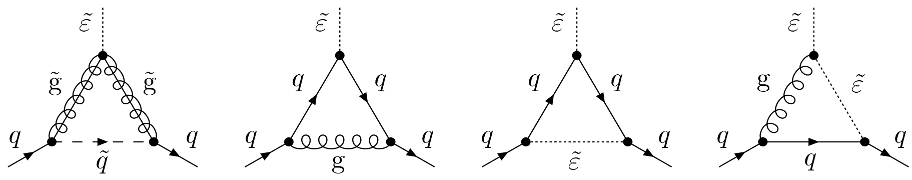

Fig. 5. One-loop diagrams for the ε-scalar Yukawa coupling renormalization constant

$$Z_{m^2_{\tilde{q}_2}} = 1 + \frac{C_F}{m^2_{\tilde{q}_2}} \frac{\alpha_s}{4\pi} \Big(\left(m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}\right) \sin^2 2\theta_{\tilde{q}} - 4 \left(m_q m_{\tilde{g}} \sin 2\theta_{\tilde{q}} + m_q^2 + m_{\tilde{g}}^2 \right) \Big) \frac{1}{\varepsilon}, \quad (48)$$

$$Z_{q\tilde{g}\tilde{q}} = 1 - \frac{\alpha_s}{4\pi} \left(\xi C_F + \frac{3+\xi}{2} C_A \right) \frac{1}{\varepsilon}, \quad (49)$$

$$Z_{\theta_{\tilde{q}}} = 1 + \frac{\delta\theta_{\tilde{q}}}{\theta_{\tilde{q}}}. \quad (50)$$

The expressions for the corresponding on-shell renormalization constants can be found in [29]. Using (24), (33), (46) and (49), one can also get the gauge coupling renormalization constant Z_{α_s} from the quark–gluino–squark vertex

$$Z_{\alpha_s} = Z^2_{q\tilde{g}\tilde{q}} Z_q^{-1} Z_{\tilde{g}}^{-1} Z_{\tilde{q}}^{-1}, \quad (51)$$

which, due to supersymmetry, of course coincides with the results of (21) and (35).

3.4 The ε-scalar sector renormalization

The renormalization of the ε-scalar sector in SUSY QCD goes along the same lines as for the ordinary QCD in dimensional reduction. Here we will recall the tricks used to obtain the corresponding renormalization constants and write down their values in SUSY QCD. The part of the SUSY QCD Lagrangian containing ε-scalar fields could be obtained through the dimensional reduction of the Lagrangian from (10). As a result we have

$$\mathcal{L}_\varepsilon = \frac{1}{2} \left(D^\mu G_\sigma \right)^\dagger D_\mu G_\sigma - g_3 \bar{q} \gamma_\sigma T^a G^a_\sigma q - \frac{1}{4} g_3^2 f^{abc} f^{ade} G^b_\sigma G^c_{\sigma'} G^d_\sigma G^e_{\sigma'} - \frac{1}{2} g_3 \bar{\tilde{g}} \gamma_\sigma T^a G^a_\sigma \tilde{g} + g_3^2 \bar{\tilde{q}} T^a T^b G^a_\sigma G^b_\sigma \tilde{q}. \quad (52)$$

Here the $G_{\sigma(\sigma')}$ denote ε-scalar fields, and the indices σ, σ' belong to the 2ε subspace.

A standard way is to use the Feynman rules derived from this Lagrangian to account for the contribution of ε-scalars. However, it turns out that for some problems one can use far simpler arguments to get a desired solution. For example, we are interested in both full and separate contributions of vectors and ε-scalars to the quantity, like a Green function or amplitude, without external vector indices. So vector and ε-scalar fields are located on internal lines only. Then to solve this problem one should

(1) perform d-dimensional vector and spinor algebras;

(2) for the full contribution: replace d in the numerator of scalarized expression with 4;

(3) for the ε-scalar contribution: replace d^n with $4^n - (4 - 2\varepsilon)^n$.

When we deal with a Green function containing external vector indices and want to extract a contribution of either external vector or ε-scalar fields, the corresponding projector onto the $4 - 2\varepsilon$ or 2ε subspaces should be constructed. Internal ε-scalars and vectors are treated in the same way, as described above. At this point one should keep in mind that particle momenta are always orthogonal to the 2ε subspace. For example, in the case of the ε-scalar propagator (see Fig. 4) we use $\frac{1}{2\varepsilon} g^{\mu\nu}$ (the ε-scalar propagator is always diagonal, see the note above) as a projector and after contraction of external indices replace d^n with $4^n - (4 - 2\varepsilon)^n$. At the same moment, evaluating corrections to the interaction between the ε-scalar and quarks (see Fig. 5), required for the determination of the Yukawa charge renormalization, one should follow the same steps, as in the case of ε-scalar propagator with the only difference that now the projector is given by

$$\mathrm{Tr} \left(\frac{1}{8 C_F C_A \varepsilon} T^a \gamma^\mu \Gamma_\mu \right), \quad (53)$$

where Γ^μ is an interaction vertex of the gluon field with quarks computed in SUSY QCD without separating the ϵ-scalar contributions, but using the four-dimensional vector and spinor algebras. In the one-loop order the ε-scalar field renormalization constant is given by

$$Z_s = 1 - \frac{\alpha_s}{4\pi} \Big(6 - C_A (2 - \xi) \Big) \frac{1}{\varepsilon}. \quad (54)$$

For the one-loop Yukawa charge renormalization constant, describing the interaction of the ϵ-scalar with fermions in

SUSY QCD, we have

$$Z_Y = Z_{qsq}^2 Z_q^{-2} Z_s^{-1} = 1 - \frac{\alpha_s}{4\pi}(3C_A - 6)\frac{1}{\varepsilon}. \qquad (55)$$

Note that in SUSY QCD, contrary to the case of non-supersymmetric QCD, it coincides with the gauge charge renormalization constant. This is due to supersymmetry, which now protects a tree-level coincidence of the ε-scalar and vector coupling constants.

The one-loop non-minimal mass counterterm for ε-scalars reads (the expression for this counterterm in the case of ordinary QCD could be found in [7])

$$\delta m_s^2 = -\frac{\alpha_s}{4\pi}\left(\sum_{n_f}\left(\frac{m_f^2}{\bar{\mu}^2}\right)^{-\varepsilon} m_f^2\left[\frac{2}{\varepsilon}+2\right]\right. \qquad (56)$$
$$\left. - \sum_{n_{\tilde{f}}}\left(\frac{m_{\tilde{f}}^2}{\bar{\mu}^2}\right)^{-\varepsilon} m_{\tilde{f}}^2\left[\frac{1}{\varepsilon}+1\right] + C_A\left(\frac{m_{\tilde{g}}^2}{\bar{\mu}^2}\right)^{-\varepsilon} m_{\tilde{g}}^2\left[\frac{2}{\varepsilon}+2\right]\right).$$

It is necessary to ensure that the pole mass of the ε-scalar is zero. Here n_f denotes the sum over different quark flavors and $n_{\tilde{f}}$ is used to denote the sum over different squarks. It is just this counterterm that cancels m_{hard}^2 terms occurring in the two-loop diagrams and ensures decoupling of large scale physics for a physical quantity like a quark pole mass. A way of renormalizing the ε-scalar mass non-minimally was first proposed in [7] and was shown to be a consistent renormalization procedure. In fact, there are two theoretically admissible prescriptions to deal with the ε-scalar mass renormalization. One is to renormalize the ε-scalar mass minimally [20] and the other is the total subtraction of loop corrections to the ε-scalar mass [7]. In the first approach, the ε-scalar mass becomes a running quantity and physical quantities, like the pole mass in our case, will in general depend on it together with other physical renormalized parameters. However, the ε-scalar mass is not a physical quantity and it is more correct to treat it as an artifact of our regularization and renormalization schemes. From this point of view the second approach is more natural, if we are going to follow as close as possible the original idea of dimensional reduction suggesting the zero tree-level mass for both ε-scalars and gauge bosons. Of course, all renormalization schemes are equivalent and could be related via a final recalculation of parameters. Introduction of a non-minimal subtraction for the ε-scalar mass leads to the fact that the bare quark mass is not anymore a renormalization group invariant. The point is that the physical quark mass depends on two mutually correlated bare masses: the quark mass itself and the ε-scalar mass. Both these quantities are $\bar{\mu}$ dependent and only the physical pole mass is RG invariant. For precisely this reason the RG equations for the bare quark mass should be written with great care.

4 Results of calculation

Here we present the results of our calculation. Corrections to the pole masses are parameterized as follows:

$$\frac{M_{\text{pole}}}{m} = 1 + \left(\frac{\Delta m}{m}\right), \qquad (57)$$

where Δm starts from the one-loop order.

Through calculations we reproduced all known results about radiative corrections to the pole masses of the bottom and top quarks: one-loop MSSM [12–16] and two-loop QCD [8,9] (in $\overline{\text{MS}}$) and [7] (in $\overline{\text{DR}}$) corrections. As an additional check, the renormalization group analysis can be applied to confirm independently the $1/\varepsilon$ and $1/\varepsilon^2$ terms. We get complete agreement between our pole terms and those from RG for the $\overline{\text{DR}}$ quark mass renormalization constant. And last but not least: calculations were performed in the linear gauge with a free QCD gauge parameter. The correction to the pole mass (57) however appears to be gauge independent.

4.1 One-loop result

The corrections to the bottom and top quark pole masses in the one-loop order have been well known for a long time for QCD as well as for MSSM. Here we reproduce them only up to terms of the order $\mathcal{O}(m_{\text{soft}}^2/m_{\text{hard}}^2)$ in a large mass expansion procedure. For pure QCD we have

$$\left(\frac{\Delta m_b}{m_b}\right)^{\text{QCD}} = C_F\frac{\alpha_s}{4\pi}\left[5 - 3\ln\left(\frac{m_b^2}{\bar{\mu}^2}\right)\right] \qquad (58)$$

and the squark–gluino MSSM contribution is given by

$$\left(\frac{\Delta m_b}{m_b}\right)^{\tilde{b}\tilde{g}} = C_F\frac{\alpha_s}{8\pi}\left[-3 + 2\ln\left(\frac{m_{\tilde{g}}^2}{\bar{\mu}^2}\right) + \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_1}^2 - m_{\tilde{g}}^2}\right.$$
$$\times\left\{\left(2 - \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_1}^2 - m_{\tilde{g}}^2} - 2\sin(2\theta_{\tilde{b}})\frac{m_{\tilde{g}}}{m_b}\right)\ln\left(\frac{m_{\tilde{b}_1}^2}{m_{\tilde{g}}^2}\right) + 1\right\}$$
$$\left. + \left(m_{\tilde{b}_1}^2 \to m_{\tilde{b}_2}^2\right)\right], \qquad (59)$$

which coincide with the results of [12–16]. To obtain similar expressions for the case of the top one should perform an obvious substitution $b \to t$. The results for squark–chargino and Higgs-quark loops are also known, but we are interested only in $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ corrections in this paper.

4.2 Two-loop result

The two-loop QCD results in the $\overline{\text{DR}}$ scheme for the b-quark (with four light quarks)

$$\left(\frac{\Delta m_b}{m_b}\right)^{\text{QCD}} = C_F\left(\frac{\alpha_s}{4\pi}\right)^2$$

$$\times \left\{-\frac{623}{18} - 8\,\zeta_2 + \ln^2\left(\frac{m_b^2}{m_t^2}\right) - \frac{13}{3}\ln\left(\frac{m_b^2}{m_t^2}\right)\right.$$
$$+ C_F\left[-\frac{59}{8} + 30\,\zeta_2 - 48\,\ln(2)\,\zeta_2 + 12\,\zeta_3 + \frac{3}{2}\ln\left(\frac{m_b^2}{\bar\mu^2}\right)\right.$$
$$\left. + \frac{9}{2}\ln^2\left(\frac{m_b^2}{\bar\mu^2}\right)\right] \quad (60)$$
$$+ C_A\left[\frac{1093}{24} - 8\,\zeta_2 + 24\,\ln(2)\,\zeta_2 - 6\,\zeta_3 - \frac{179}{6}\ln\left(\frac{m_b^2}{\bar\mu^2}\right)\right.$$
$$\left.\left. + \frac{11}{2}\ln^2\left(\frac{m_b^2}{\bar\mu^2}\right)\right] + 26\,\ln\left(\frac{m_b^2}{\bar\mu^2}\right) - 6\ln^2\left(\frac{m_b^2}{\bar\mu^2}\right)\right\},$$

and for the t-quark (with five light quarks)

$$\left(\frac{\Delta m_t}{m_t}\right)^{\rm QCD} = C_F\left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left\{-43 - 12\,\zeta_2 + 26\,\ln\left(\frac{m_t^2}{\bar\mu^2}\right) - 6\ln^2\left(\frac{m_t^2}{\bar\mu^2}\right)\right.$$
$$+ C_F\left[-\frac{59}{8} + 30\,\zeta_2 - 48\,\ln(2)\,\zeta_2 + 12\,\zeta_3 + \frac{3}{2}\ln\left(\frac{m_t^2}{\bar\mu^2}\right)\right.$$
$$\left. + \frac{9}{2}\ln^2\left(\frac{m_t^2}{\bar\mu^2}\right)\right]$$
$$+ C_A\left[\frac{1093}{24} - 8\,\zeta_2 + 24\,\ln(2)\,\zeta_2 - 6\,\zeta_3 - \frac{179}{6}\ln\left(\frac{m_t^2}{\bar\mu^2}\right)\right.$$
$$\left.\left. + \frac{11}{2}\ln^2\left(\frac{m_t^2}{\bar\mu^2}\right)\right]\right\}$$

coincide with the first terms of the expansion in m_b/m_t from [7]

We find a very simple answer to one particular limit. Namely, in the case, when $m_{\tilde t_{\rm L}} = m_{\tilde t_{\rm R}} = m_{\tilde b_{\rm L}} = m_{\tilde b_{\rm R}} = m_{\tilde g} = m_{\tilde u_{1,2}} = m_{\tilde d_{1,2}} = m_{\rm SUSY} = M$, the expression looks like (see (12) for the definition of a_q)

$$\left(\frac{\Delta m_q}{m_q}\right)^{\rm MSSM} = C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{47}{3} + 20\ln\left(\frac{M^2}{\bar\mu^2}\right)\right.$$
$$+ 6\ln\left(\frac{M^2}{\bar\mu^2}\right)\ln\left(\frac{M^2}{m_q^2}\right)$$
$$+ C_F\left[\frac{23}{24} - \frac{13}{6}\ln\left(\frac{M^2}{\bar\mu^2}\right) + \frac{1}{2}\ln^2\left(\frac{M^2}{\bar\mu^2}\right)\right.$$
$$\left. - 3\ln\left(\frac{M^2}{\bar\mu^2}\right)\ln\left(\frac{m_q^2}{\bar\mu^2}\right)\right]$$
$$+ C_A\left[\frac{175}{72} + \frac{41}{6}\ln\left(\frac{M^2}{\bar\mu^2}\right) - \frac{1}{2}\ln^2\left(\frac{M^2}{\bar\mu^2}\right)\right.$$
$$\left. - 2\ln\left(\frac{M^2}{\bar\mu^2}\right)\ln\left(\frac{m_q^2}{\bar\mu^2}\right)\right] + \frac{a_q}{M}\left[-4 - 8\ln\left(\frac{M^2}{\bar\mu^2}\right)\right]$$
$$+ C_A\frac{a_q}{M}\left[-\frac{8}{3} + 4\ln\left(\frac{M^2}{\bar\mu^2}\right)\right]$$
$$\left. + C_F\frac{a_q}{M}\left[\frac{7}{3} - \frac{11}{3}\ln\left(\frac{M^2}{\bar\mu^2}\right) + 3\ln\left(\frac{m_q^2}{\bar\mu^2}\right)\right]\right\}. \quad (61)$$

Note that we put equal to each other the left and right squark masses $m^2_{\tilde q_{\rm R}} = m^2_{\tilde q_{\rm L}}$ but not the physical mass eigenstates $m^2_{\tilde q_1} = m^2_{\tilde q_2}$. This latter equation is true only when $a_q = 0$ because a minimal mixing take place when $\theta_q = \pi/4$ and $m^2_{\tilde q_{1,2}} = M^2 \pm a_q m_q$ (for $a_q < 0$).

Since the complete formulae[8] are too large to be presented here, we present only the influence of our results on the masses of the b- and t-quarks and the spectra of supersymmetric particles obtained as solutions of RG equations.

We incorporated our formulae into the SoftSUSY code [6] and analyzed the difference in the predictions for particle masses obtained with and without our corrections to b- and t-quark pole masses. To measure quantitatively the influence on the spectra let us introduce for each particle species the following quantity:

$$\Delta_p = \frac{m_p^{\rm CODE} - m_p^{\rm CODE+Corrections}}{m_p^{\rm CODE}}, \quad (62)$$

where $m_p^{\rm CODE}$ is the mass of the particle p (here p stands for the particle of SM or its possible superpartners) obtained with the use of the original code of the SoftSUSY program and $m_p^{\rm CODE+Corrections}$ is the mass of the same particle obtained using the code including our two-loop corrections. Below we set in our analysis the renormalization scale $\bar\mu$ equal to m_Z.

In Fig. 6, we present the variation of Δ_p from the above formula as a function of $\tan\beta$ for $m_0 = 400$ GeV, $m_{1/2} = 400$ GeV, $A_0 = 0$ and $M_{\rm GUT} = 1.9\times10^{16}$ GeV. We plotted $\Delta m_p/m_p$ for five different cases:

(i) *Bottom 1L*: difference between the results obtained with code including and excluding MSSM one-loop corrections to the pole mass of the b-quark;

(ii) *Top 1L*: the same as above for the t-quark;

(iii) *Bottom*: the difference between the results obtained with the code containing a two-loop SUSY QCD correction to the b-quark together with MSSM one-loop corrections to the pole masses of b- and t-quarks and results obtained with the code containing MSSM one-loop corrections to the pole masses of b- and t-quarks;

(iv) *Top*: the same as above for the t-quark;

(v) *All*: the difference between the results obtained with the code containing two-loop SUSY QCD correction together with MSSM one-loop corrections to the pole masses of b- and t-quarks and the results obtained with the code without any MSSM (one-loop and two-loop) corrections to the pole masses of b- and t-quarks.

From this figure one can see that the two-loop correction to the relation between the b-quark pole and $\overline{\rm DR}$ masses gives a sizable contribution only to the solution obtained with the SoftSUSY program for the $\overline{\rm DR}$ b-quark mass at m_Z and almost does not influence the spectra of the SUSY particles. However, if one considers a large $\tan\beta$ it becomes important for other particle spectra and thus

[8] They can be found in the source archive file for this paper submitted to the HEP database

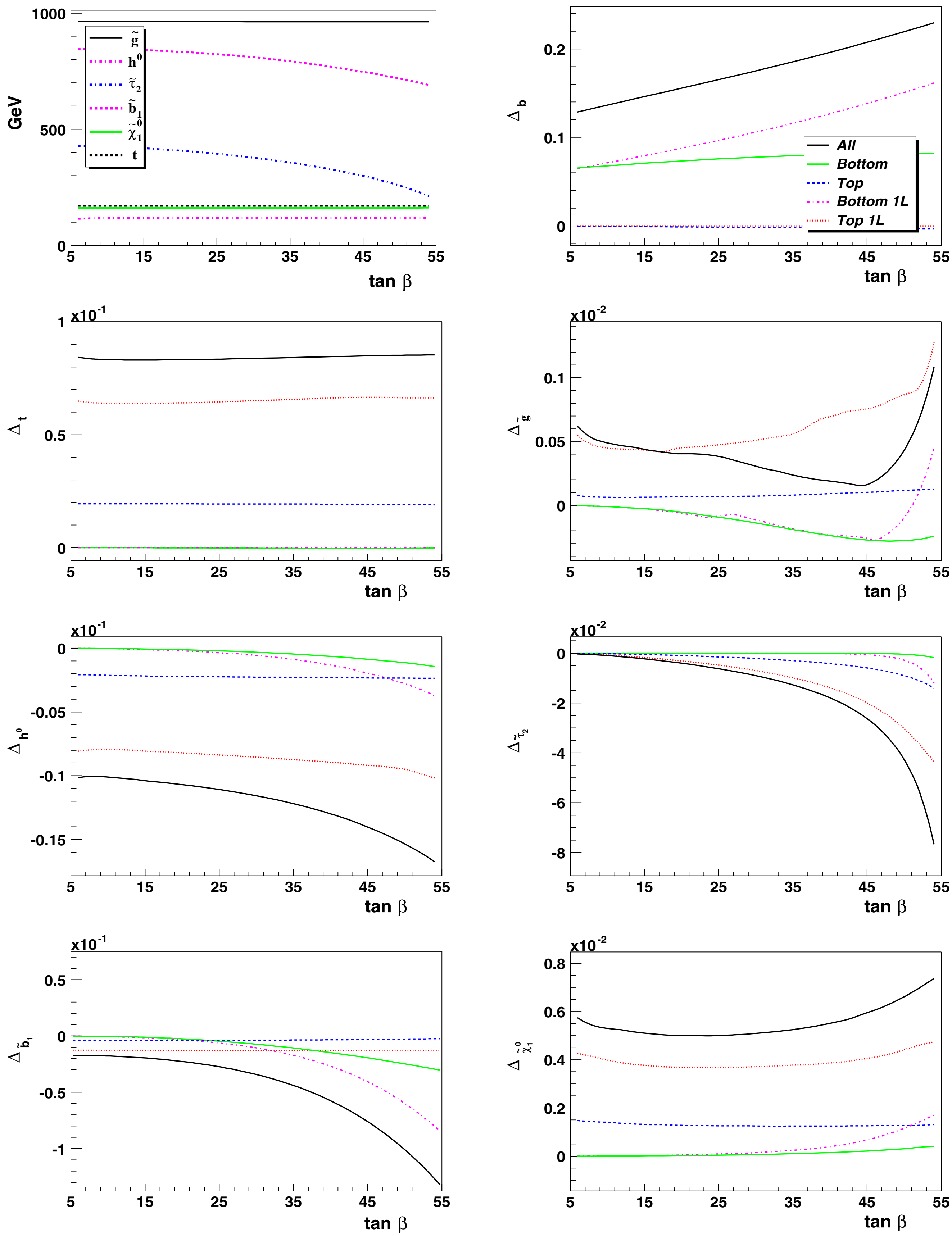

Fig. 6. Δ_p from the formula (62) as functions of $\tan\beta$. In the first picture we plotted a spectrum of masses (see also explanation in the text)

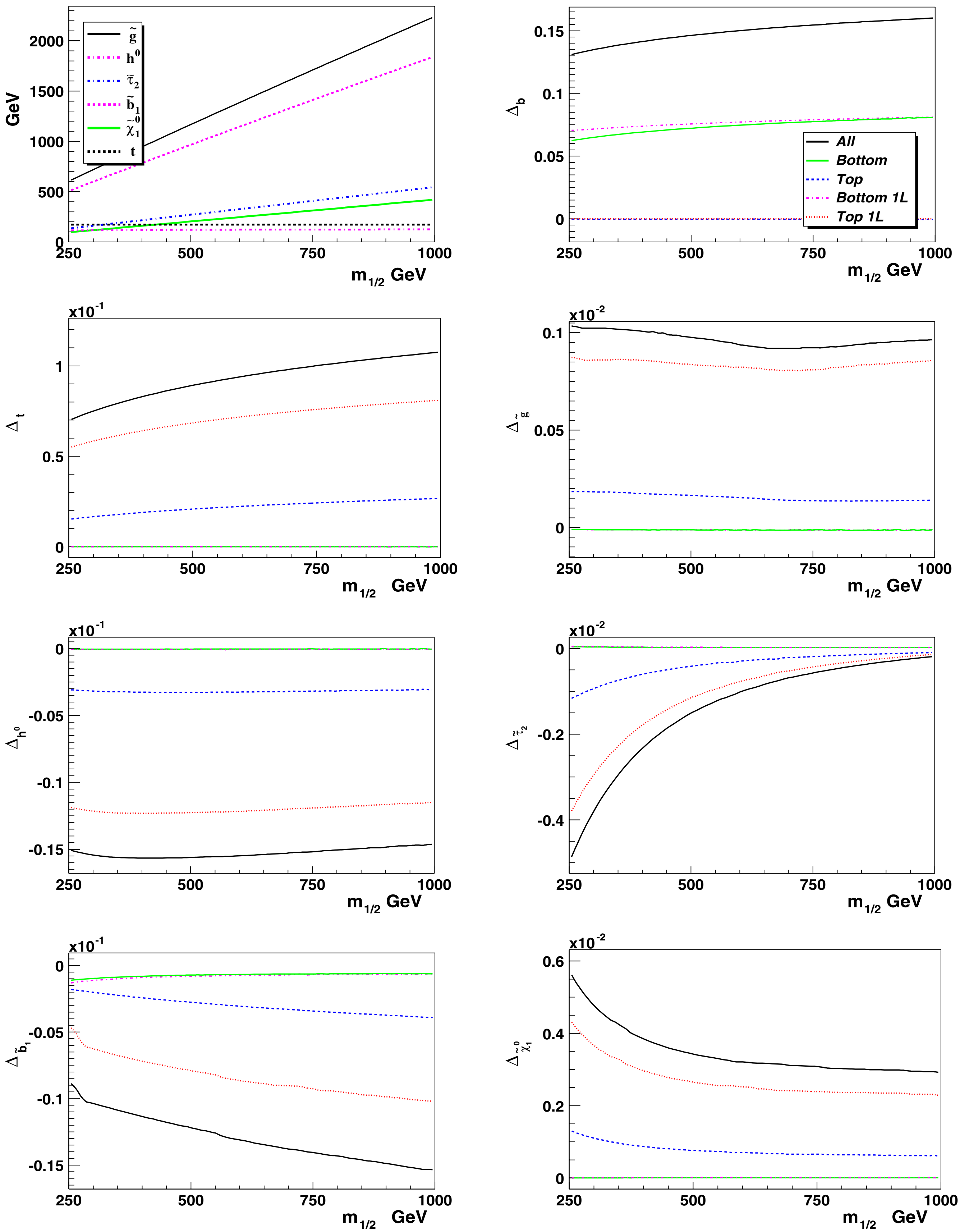

Fig. 7. Δ_p from the formula (62) as functions defined along *Model Line A* from corrections depending on $m_{1/2}$. In the first picture we plotted a spectrum of the masses

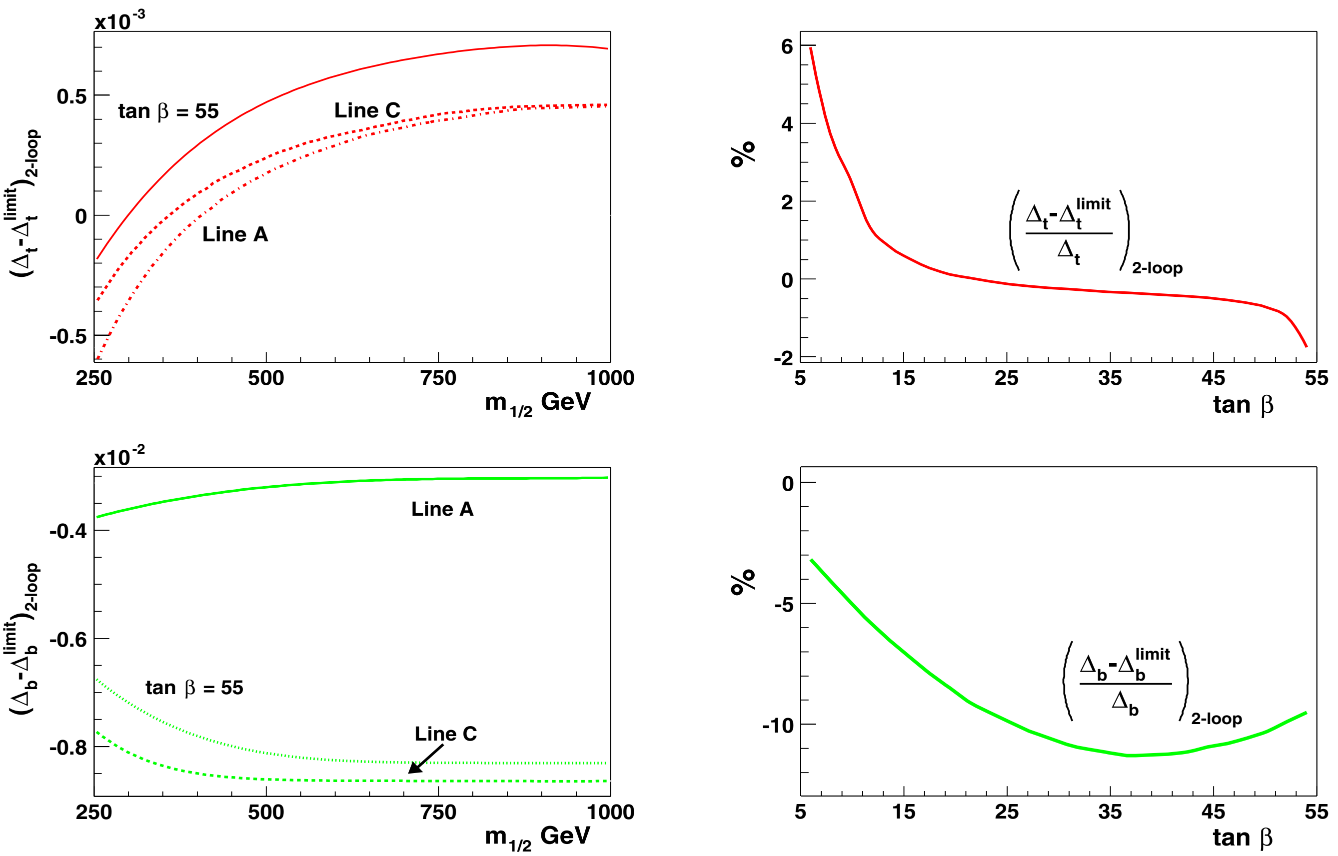

Fig. 8. Difference between Δ_p with full two-loop results and limit from (61). Upper plots are for the t-quark and the bottom ones are for the b-quark. Plots to the left show the dependence on $m_{1/2}$ of the difference between Δ_p and Δ_p^{limit} for *Model Line A*, *Model Line C* and for $\tan\beta = 55$. Plots to the right show the relative dependence on $\tan\beta$ of the difference between Δ_p and Δ_p^{limit} for the same parameters as in the text

should be taken into account in an accurate RG analysis. Note that our two-loop SUSY QCD correction for a wide range of parameter space is always less than a similar one-loop contribution. The two-loop MSSM correction to the t-quark pole mass, on the other hand, contributes to almost all SUSY particle masses obtained as a solution of the renormalization group equations with universal boundary conditions at the GUT scale. This two-loop correction is smaller than the one-loop MSSM contribution; however its effect on the particle spectra is greater than the corresponding one- and two-loop MSSM contributions to the b-quark pole mass almost everywhere in MSSM parameter space. This could be attributed to potentially large corrections coming from higher terms in the m_t/M_{SUSY} expansion.

To see the dependence of Δ_p on m_0, $m_{1/2}$ and A_0, we use a so-called set of benchmark points and parameter lines in MSSM parameter space from [30–33] which corresponds to different scenarios in the search for supersymmetry at present and future colliders. While a detailed scan over the more-than-hundred-dimensional parameter space of MSSM is clearly not practicable, even a sampling of the three- (four-) dimensional parameter space of m_0, $m_{1/2}$ and A_0 ($\tan\beta$) is beyond the present capabilities of phenomenological studies, especially when one tries to simulate experimental signatures of supersymmetric particles within detectors. For this reason, one often resorts to specific benchmark scenarios, i.e., one studies only specific parameter points or at best one samples the one-dimensional parameter space (the latter is sometimes called a model line [32]), which exhibits specific characteristics of MSSM parameter space.

Some recent proposals for SUSY benchmark scenarios may be found in [30,31,33]. We refer to the "snowmass points and slopes" (SPS) [32,33] which consists of model lines ("slopes"), i.e., continuous sets of parameters depending on the one dimensionful parameter and specific benchmark points, where each model line goes through one of the benchmark points.

In Fig. 7, we present the results for Δ_p as functions defined along *Model Line A*

$$m_0 = -A_0 = 0.4\, m_{1/2}, \quad m_{1/2} \text{ varies}, \quad \tan\beta = 10, \quad \mu > 0.$$

where the benchmark point is

$$m_0 = 100\,\text{GeV}, \quad m_{1/2} = 250\,\text{GeV}, \quad A_0 = -100\,\text{GeV}.$$

Here and below μ is the supersymmetric Higgs mass parameter.

As one can easily see from Fig. 7, varying $m_{1/2}$ at fixed $\tan\beta$ the two-loop SUSY QCD correction to the b-quark pole mass almost does not contribute to heavy supersymmetric particle spectra. On the other hand, our correction to the t-quark pole mass is comparable to the corresponding one-loop contribution.

Also we have seen the same behavior along *Model Line C*:

$$m_0 = m_{1/2}, \quad m_{1/2} \text{ varies}, \quad A_0 = 0, \quad \tan\beta = 35, \quad \mu > 0.$$

where the benchmark point is

$$m_0 = 300\,\mathrm{GeV}, \quad m_{1/2} = 300\,\mathrm{GeV}, \quad A_0 = 0\,\mathrm{GeV}.$$

The situation in this case is analogous to that of Fig. 7, even if we increase $\tan\beta$ from 10 up to 35. The importance of our corrections increases only at very large values of $\tan\beta$ (50 and more), as one can see from Fig. 6.

Now let us discuss our limiting formula (61). We have studied the difference between our full results and this approximate expression in different regions of MSSM parameter space using the same strategy as above. As a result we find (see Fig. 8) that for the b-quark this difference does not exceed 12% for the parametrs region which we see, both for the dependence on $\tan\beta$ and $m_{1/2}$. Here, we would like to note that the limiting formula in the case of the b-quark works better for small or very large values of $\tan\beta$ and for small (large for *Model Line A*) values of $m_{1/2}$. In the case of the t-quark, on the contrary, it works better for middle values of $\tan\beta$ and $m_{1/2}$.

5 Conclusion

We presented the results for two-loop MSSM corrections to the relation between pole and running masses of heavy quarks up to the $\mathcal{O}(\alpha_s^2)$ order. We provided a detailed analysis of the value of these corrections in different regions of parameter space and discussed their impact on the SUSY particle spectra. We would like to note that our results presented here may also be included in the codes of programs like Isajet [34], SuSpect [35] and used for predictive phenomenological analysis like [36–39]. In one of our next papers we propose to calculate missing corrections from stop–chargino loops to the b-quark pole mass and to provide more terms in the expansion in the relation between the top quark pole and $\overline{\mathrm{DR}}$ masses.

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